SOUTH-WESTERN CENGAGE Learning

CHAPTER 11

SOLUTIONS TO PROBLEMS

11.1 Because of covariance stationarity, γ_0 = Var(x_t) does not depend on t, so sd(x_{t+h}) = $\sqrt{\gamma_0}$ for any $h \geq 0$. By definition, Corr(x_t,x_{t+h}) = Cov(x_t,x_{t+h})/[sd(x_t)$\cdot$ sd(x_{t+h})] = $\gamma_h / (\sqrt{\gamma_0} \cdot \sqrt{\gamma_0}) = \gamma_h / \gamma_0$.

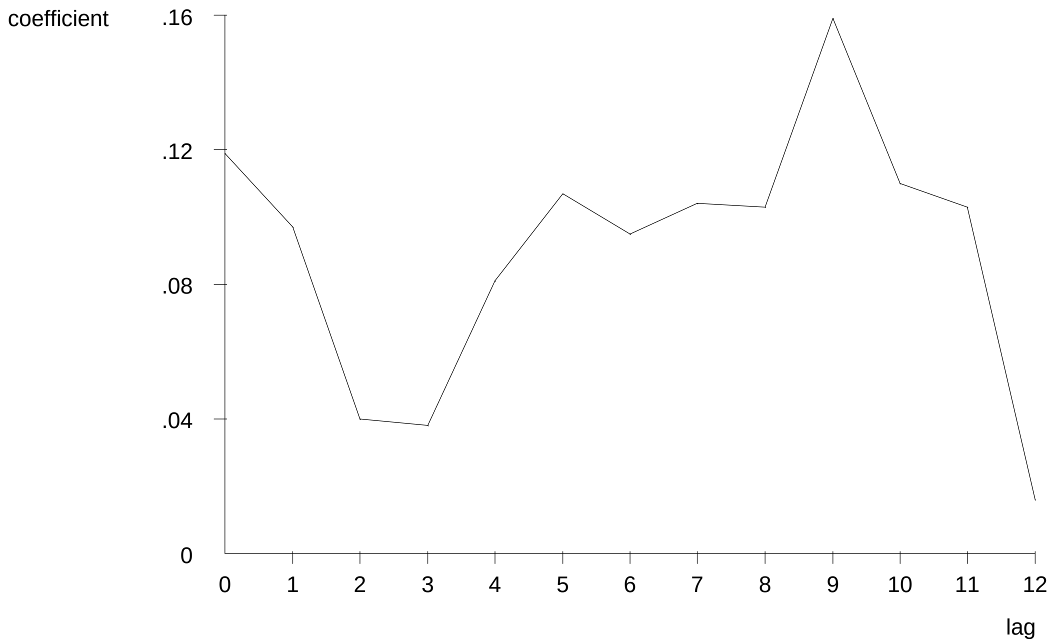
11.3 (i) E(y_t) = E(z + e_t) = E(z) + E(e_t) = 0. Var(y_t) = Var(z + e_t) = Var(z) + Var(e_t) + 2Cov(z,e_t) = $\sigma_z^2 + \sigma_e^2 + 2 \cdot 0 = \sigma_z^2 + \sigma_e^2$. Neither of these depends on t.

(ii) We assume $h > 0$; when $h = 0$ we obtain Var(y_t). Then Cov(y_t,y_{t+h}) = E($y_t y_{t+h}$) = E[(z + e_t)(z + e_{t+h})] = E(z^2) + E(ze_{t+h}) + E($e_t z$) + E($e_t e_{t+h}$) = E(z^2) = σ_z^2 because $\{e_t\}$ is an uncorrelated sequence (it is an independent sequence and z is uncorrelated with e_t for all t. From part (i) we know that E(y_t) and Var(y_t) do not depend on t and we have shown that Cov(y_t,y_{t+h}) depends on neither t nor h. Therefore, $\{y_t\}$ is covariance stationary.

(iii) From Problem 11.1 and parts (i) and (ii), Corr(y_t,y_{t+h}) = Cov(y_t,y_{t+h})/Var(y_t) = $\sigma_z^2 / (\sigma_z^2 + \sigma_e^2) > 0$.

(iv) No. The correlation between y_t and y_{t+h} is the same positive value obtained in part (iii) now matter how large is h. In other words, no matter how far apart y_t and y_{t+h} are, their correlation is always the same. Of course, the persistent correlation across time is due to the presence of the time-constant variable, z.

11.5 (i) The following graph gives the estimated lag distribution:



By some margin, the largest effect is at the ninth lag, which says that a temporary increase in wage inflation has its largest effect on price inflation nine months later. The smallest effect is at the twelfth lag, which hopefully indicates (but does not guarantee) that we have accounted for enough lags of *gwage* in the FLD model.

(ii) Lags two, three, and twelve have *t* statistics less than two. The other lags are statistically significant at the 5% level against a two-sided alternative. (Assuming either that the CLM assumptions hold for exact tests or Assumptions TS.1′ through TS.5′ hold for asymptotic tests.)

(iii) The estimated LRP is just the sum of the lag coefficients from zero through twelve: 1.172. While this is greater than one, it is not much greater, and the difference from unity could be due to sampling error.

(iv) The model underlying and the estimated equation can be written with intercept α_0 and lag coefficients $\delta_0, \delta_1, \dots, \delta_{12}$. Denote the LRP by $\theta_0 = \delta_0 + \delta_1 + \dots + \delta_{12}$. Now, we can write $\delta_0 = \theta_0 - \delta_1 - \delta_2 - \dots - \delta_{12}$. If we plug this into the FDL model we obtain (with $y_t = gprice_t$ and $z_t = gwage_t$)

$$\begin{aligned} y_t &= \alpha_0 + (\theta_0 - \delta_1 - \delta_2 - \dots - \delta_{12})z_t + \delta_1 z_{t-1} + \delta_2 z_{t-2} + \dots + \delta_{12} z_{t-12} + u_t \\ &= \alpha_0 + \theta_0 z_t + \delta_1(z_{t-1} - z_t) + \delta_2(z_{t-2} - z_t) + \dots + \delta_{12}(z_{t-12} - z_t) + u_t. \end{aligned}$$

Therefore, we regress y_t on z_t, $(z_{t-1} - z_t)$, $(z_{t-2} - z_t)$, …, $(z_{t-12} - z_t)$ and obtain the coefficient and standard error on z_t as the estimated LRP and its standard error.

(v) We would add lags 13 through 18 of $gwage_t$ to the equation, which leaves 273 – 6 = 267 observations. Now, we are estimating 20 parameters, so the df in the unrestricted model is df_{ur} = 267. Let R_{ur}^2 be the R-squared from this regression. To obtain the restricted R-squared, R_r^2, we need to reestimate the model reported in the problem but with the same 267 observations used to estimate the unrestricted model. Then $F = [(R_{ur}^2 - R_r^2)/(1 - R_{ur}^2)](247/6)$. We would find the critical value from the $F_{6,247}$ distribution.

11.7 (i) We plug the first equation into the second to get

$$y_t - y_{t-1} = \lambda(\gamma_0 + \gamma_1 x_t + e_t - y_{t-1}) + a_t,$$

and, rearranging,

$$y_t = \lambda\gamma_0 + (1 - \lambda)y_{t-1} + \lambda\gamma_1 x_t + a_t + \lambda e_t,$$

$$\equiv \beta_0 + \beta_1 y_{t-1} + \beta_2 x_t + u_t,$$

where $\beta_0 \equiv \lambda\gamma_0$, $\beta_1 \equiv (1 - \lambda)$, $\beta_2 \equiv \lambda\gamma_1$, and $u_t \equiv a_t + \lambda e_t$.

(ii) An OLS regression of y_t on y_{t-1} and x_t produces consistent, asymptotically normal estimators of the β_j. Under $E(e_t|x_t,y_{t-1},x_{t-1}, \dots) = E(a_t|x_t,y_{t-1},x_{t-1}, \dots) = 0$ it follows that $E(u_t|x_t,y_{t-1},x_{t-1}, \dots) = 0$, which means that the model is dynamically complete [see equation (11.37)]. Therefore, the errors are serially uncorrelated. If the homoskedasticity assumption $Var(u_t|x_t,y_{t-1}) = \sigma^2$ holds, then the usual standard errors, t statistics and F statistics are asymptotically valid.

(iii) Because $\beta_1 = (1 - \lambda)$, if $\hat{\beta}_1 = .7$ then $\hat{\lambda} = .3$. Further, $\hat{\beta}_2 = \hat{\lambda}\hat{\gamma}_1$, or $\hat{\gamma}_1 = \hat{\beta}_2/\hat{\lambda} = .2/.3 \approx .67$.

SOLUTIONS TO COMPUTER EXERCISES

C11.1 (i) The first order autocorrelation for log($invpc$) is about .639. If we first detrend log($invpc$) by regressing on a linear time trend, $\hat{\rho}_1 \approx .485$. Especially after detrending there is little evidence of a unit root in log($invpc$). For log($price$), the first order autocorrelation is about .949, which is very high. After detrending, the first order autocorrelation drops to .822, but this is still pretty large. We cannot confidently rule out a unit root in log($price$).

(ii) The estimated equation is

$$\widehat{\log(invpc_t)} = -.853 + 3.88\,\Delta\log(price_t) + .0080\,t$$
$$\quad\quad\;\; (.040) \quad (0.96) \quad\quad\quad\quad\quad (.0016)$$
$$n = 41,\;\; R^2 = .501.$$

The coefficient on $\Delta\log(price_t)$ implies that a one percentage point increase in the growth in price leads to a 3.88 percent increase in housing investment above its trend. [If $\Delta\log(price_t) = .01$ then $\Delta\widehat{\log(invpc_t)} = .0388$; we multiply both by 100 to convert the proportionate changes to percentage changes.]

(iii) If we first linearly detrend $\log(invpc_t)$ before regressing it on $\Delta\log(price_t)$ and the time trend, then $R^2 = .303$, which is substantially lower than that when we do not detrend. Thus, $\Delta\log(price_t)$ explains only about 30% of the variation in $\log(invpc_t)$ about its trend.

(iv) The estimated equation is

$$\Delta\widehat{\log(invpc_t)} = .006 + 1.57\,\Delta\log(price_t) + .00004t$$
$$\quad\quad\;\; (.048) \quad (1.14) \quad\quad\quad\quad\quad (.00190)$$
$$n = 41,\;\; R^2 = .048.$$

The coefficient on $\Delta\log(price_t)$ has fallen substantially and is no longer significant at the 5% level against a positive one-sided alternative. The R-squared is much smaller; $\Delta\log(price_t)$ explains very little variation in $\Delta\log(invpc_t)$. Because differencing eliminates linear time trends, it is not surprising that the estimate on the trend is very small and very statistically insignificant.

C11.3 (i) The estimated equation is

$$\widehat{return_t} = .226 + .049\; return_{t-1} - .0097\; return_{t-1}^2$$
$$\quad\quad (.087) \quad\quad (.039) \quad\quad\quad\quad (.0070)$$
$$n = 689,\;\; R^2 = .0063.$$

(ii) The null hypothesis is H_0: $\beta_1 = \beta_2 = 0$. Only if both parameters are zero does $E(return_t|return_{t-1})$ not depend on $return_{t-1}$. The F statistic is about 2.16 with p-value $\approx$ .116. Therefore, we cannot reject H_0 at the 10% level.

(iii) When we put $return_{t-1}\cdot return_{t-2}$ in place of $return_{t-1}^2$ the null can still be stated as in part (ii): no past values of *return*, or any functions of them, should help us predict $return_t$. The R-squared is about .0052 and $F \approx 1.80$ with p-value $\approx$ .166. Here, we do not reject H_0 at even the 15% level.

(iv) Predicting $return_t$ based on past returns does not appear promising. Even though the F statistic from part (ii) is almost significant at the 10% level, we have many observations. We cannot even explain 1% of the variation in $return_t$.

C11.5 (i) The estimated equation is

$$\Delta\widehat{gfr} = -1.27 - .035\,\Delta pe - .013\,\Delta pe_{-1} - .111\,\Delta pe_{-2} + .0079\,t$$
$$\quad\;(1.05)\quad(.027)\qquad(.028)\qquad\quad(.027)\qquad\quad(.0242)$$

$$n = 69,\quad R^2 = .234,\quad \bar{R}^2 = .186.$$

The time trend coefficient is very insignificant, so it is not needed in the equation.

(iii) The estimated equation is

$$\Delta\widehat{gfr} = -.650 - .075\,\Delta pe - .051\,\Delta pe_{-1} + .088\,\Delta pe_{-2} + 4.84\,ww2 - 1.68\,pill$$
$$\quad(.582)\quad(.032)\qquad(.033)\qquad\quad(.028)\qquad\quad(2.83)\qquad(1.00)$$

$$n = 69,\quad R^2 = .296,\quad \bar{R}^2 = .240.$$

The F statistic for joint significance is $F = 2.82$ with p-value $\approx .067$. So $ww2$ and $pill$ are not jointly significant at the 5% level, but they are at the 10% level.

(iii) By regressing Δgfr on Δpe, $(\Delta pe_{-1} - \Delta pe)$. $(\Delta pe_{-2} - \Delta pe)$, $ww2$, and $pill$, we obtain the LRP and its standard error as the coefficient on Δpe: $-.075$, se = .032. So the estimated LRP is now negative and significant, which is very different from the equation in levels, (10.19) (the estimated LRP was .101 with a t statistic of about 3.37). This is a good example of how differencing variables before including them in a regression can lead to very different conclusions than a regression in levels.

C11.7 (i) If $E(gc_t|I_{t-1}) = E(gc_t)$ – that is, $E(gc_t|I_{t-1})$ = does not depend on gc_{t-1}, then $\beta_1 = 0$ in $gc_t = \beta_0 + \beta_1 gc_{t-1} + u_t$. So the null hypothesis is H_0: $\beta_1 = 0$ and the alternative is H_1: $\beta_1 \neq 0$. Estimating the simple regression using the data in CONSUMP.RAW gives

$$\widehat{gc}_t = .011 + .446\,gc_{t-1}$$
$$\qquad(.004)\quad(.156)$$

$$n = 35,\quad R^2 = .199.$$

The t statistic for $\hat{\beta}_1$ is about 2.86, and so we strongly reject the PIH. The coefficient on gc_{t-1} is also practically large, showing significant autocorrelation in consumption growth.

(ii) When gy_{t-1} and $i3_{t-1}$ are added to the regression, the R-squared becomes about .288. The F statistic for joint significance of gy_{t-1} and $i3_{t-1}$, obtained using the Stata “test” command, is

1.95, with p-value ≈ .16. Therefore, gy_{t-1} and $i3_{t-1}$ are not jointly significant at even the 15% level.

C11.9 (i) The first order autocorrelation for *prcfat* is .709, which is high but not necessarily a cause for concern. For *unem*, $\hat{\rho}_1 = .950$, which is cause for concern in using *unem* as an explanatory variable in a regression.

(ii) If we use the first differences of *prcfat* and *unem*, but leave all other variables in their original form, we get the following:

$$\Delta\widehat{prcfat} = \underset{(.105)}{-.127} + \ldots + \underset{(.0072)}{.0068}\, wkends + \underset{(.0161)}{.0125}\, \Delta unem$$

$$- \underset{(.0238)}{.0072}\, spdlaw + \underset{(.0265)}{.0008}\, bltlaw$$

$n = 107$, $R^2 = .344$,

where I have again suppressed the coefficients on the time trend and seasonal dummies. This regression basically shows that the change in *prcfat* cannot be explained by the change in *unem* or any of the policy variables. It does have some seasonality, which is why the R-squared is .344.

(iii) This is an example about how estimation in first differences loses the interesting implications of the model estimated in levels. Of course, this is not to say the levels regression is valid. But, as it turns out, we can reject a unit root in *prcfat*, and so we can at least justify using it in level form; see Computer Exercise 18.13. Generally, the issue of whether to take first differences is very difficult, even for professional time series econometricians.

C11.11 (i) The estimated equation is

$$\widehat{pcrgdp}_t = \underset{(0.163)}{3.344} - \underset{(0.182)}{1.891}\, \Delta unem_t$$

$n = 46$, $R^2 = .710$

Naturally, we do not get the exact estimates specified by the theory. Okun's Law is expected to hold, at best, on average. The estimates are not particularly far from their hypothesized values of 3 (intercept) and −2 (slope).

(ii) The t statistic for testing $H_0: \beta_1 = -2$ is about .60, which gives a two-sided p-value of about .55. This is very little evidence against H_0; the null is not rejected at any reasonable significance level.

(iii) The *t* statistic for $H_0: \beta_0 = 3$ is about 2.11, and the two-sided *p*-value is about .04. Therefore, the null is rejected at the 5% level, although it is not much stronger than that.

(iv) The joint test underlying Okun's Law gives *F* = 2.41. With (2,44) *df*, we get, roughly, *p*-value = .10. Therefore, Okun's Law passes at the 5% level, but only just at the 10% level.